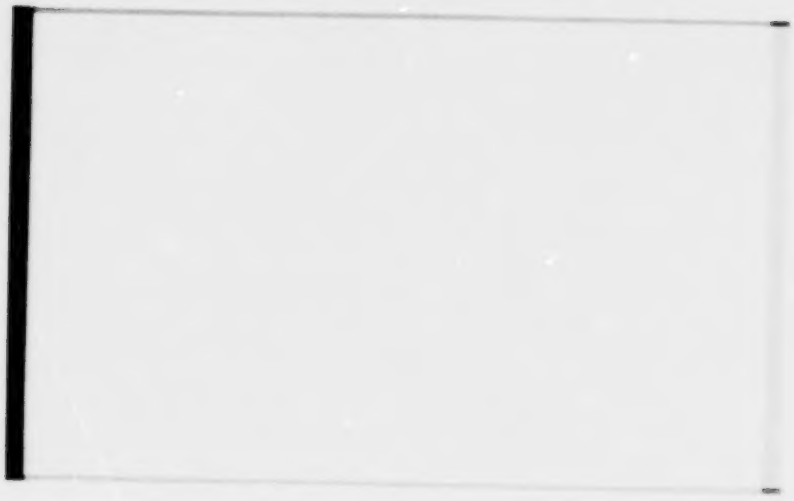
UNIVERSITÉ DE MONTRÉAL

CENTRE DE RECHERCHE SUR LES TRANSPORTS

CASE POSTALE 6128, SUCCURSALE A, MONTRÉAL, QUÉBEC, H3C 3J7

88-03984/



STOCHASTIC MODELS FOR BULK INVENTORIES IN TRANSPORTATION TERMINALS*

by

Nelio Domingues Pizzolato[1]
and
Jean Ethier[2]

* The pratical applications in this study were made possible by the Quebec Cartier Mining Company who supplied us with the data.

[1] Associate Professor, Industrial Engineering Department, Universidade Catolica de Rio de Janeiro, Brazil, on sabbatical leave at the Centre de recherche sur les transports, Université de Montréal, supported by a grant from the Conselho Nacional de Desenvolvimento Cientifico e Tecnologico, CNPq, Process no. 20.2672/85.

[2] Associate Professional, G.E.R.A.D., Université de Montréal.

Cette étude a été publiée grâce à une subvention du fonds F.C.A.R. pour la l'aide et le soutien à recherche.

Université de Montréal
Centre de recherche sur les transports - Publication #565
March 1988

Dépôt légal - 1er trimestre 1988

ABSTRACT

This research examines the problem of mathematical modelling of bulk inventories in transportation terminals given two arrival distributions: trains that resupply and ships that deplete the terminal. Two sets of models are proposed, both based on Markov chains. The first simply forecasts inventory levels. The second helps designing the terminal assuming it is composed of two sectors, a main and a reserve one which handles at a lower cost the excess of stock. Both models are applied to real situations encountered by the Quebec Cartier iron ore terminal and it is shown that they bring a contribution to the design of terminals.

Key words: transportation terminals, Markov chains, inventory control.

SOMMAIRE

Ce travail étudie le problème de modélisation mathématique d'inventaires en vrac dans les terminaux de transport, étant donné deux distributions d'arrivée des produits: les trains qui remplissent le terminal et les bateaux qui le vident. Deux modèles, basés sur la théorie des chaînes de Markov, sont proposés ici. Le premier effectue simplement la prévision des niveaux d'inventaire. Le second sert à la planification du terminal, en supposant que ce dernier est divisé en deux parties: une partie principale et une réserve dont le coût de manipulation de l'excès de stock est peu élevé. Les deux modèles ont été appliqués à des situations réelles rencontrées par la compagnie minière Québec Cartier et les résultats démontrent que ces modèles contribuent à la planification des terminaux.

Mots clés: terminaux de transport, chaînes de Markov, contrôle d'inventaire.

TABLE OF CONTENTS

I.	INTRODUCTION	1
II.	THE SINGLE STORAGE MODEL	3
	2.1 DETERMINATION OF TOTAL SHIP CAPACITY	5
	2.2 THE TRANSITION MATRIX	9
	2.3 CRITIQUE OF THE ASSUMPTIONS	12
	2.4 THE SHORT-TERM DECISION PROCESS	14
III.	A REAL APPLICATION	16
	3.1 TRAINS	16
	3.2 SHIPS	20
	3.3 SOLUTION OF THE SYSTEM	24
	3.4 EVALUATION OF THE RESULTS	28
	3.5 CONCLUSIONS	30
IV.	THE DOUBLE-STORAGE MODEL	32
	4.1 THE TWO-DIMENSIONAL STATE MODEL	34
	4.2 THE TWO-REFLECTING STATES MODEL	37
	4.3 THE ONE-REFLECTING STATE MODEL	39
V.	A PRACTICAL APPLICATION	44
	5.1 SOLUTION OF THE SYSTEM	47
	5.2 THE CONVEYOR OPERATION	51
	5.3 CONCLUSIONS	55
	BIBLIOGRAPHY	56
	ACKNOWLEDGMENTS	56

I. INTRODUCTION

Bulk terminals are facilities specialized in handling large amounts of a unique class of product. Thus, cereal terminals, salt terminals, iron ore terminals, etc. are usual denominations derived from the product they were projected for. Terminals may exist at any point in which the transport mode is changed but are especially important in the case of sea transportation where they may accumulate sizable amounts of product that arrive by train or truck to be dispatched in large vessels.

The design of any terminal requires the existence of three specialized but interconnected systems: i) reception system, including discharging equipment and controls upon volume, weight and quality of incoming product, ii) storage system, including materials handling equipment, protection, safety, and inventory controls, and iii) dispatching system composed of recuperation and transportation equipment with control on flows, volumes, weights, and quality of departing product.

Queueing theory is an analytical instrument normally required to support the design of any terminal. It considers two separate and independent queueing processes: one at the reception system and the other at the dispatching system, and it produces answers to a number of questions such as the optimal number of parallel discharging and charging equipments, the choice and effect of servicing speeds, the waiting time and queue size for each transporting mode, the busy and the idle probability for the operating equipment, etc.

There are, however, a number of interesting questions remaining that are unlikely or not at all examined by queueing theory such as the design of storage capacity, and other important stock-level related questions. The present paper addresses two aspects of the inventory problem and, therefore,

has two broad but distinct objectives. In both cases, discrete Markov decision models in which the system is observed at the end of equally spaced periods such as the day, the week, etc, are proposed.

The first objective is the development of a stochastic model to forecast stock levels resulting from the two interfacing queues. For a given maximum storage capacity, the steady state solution will tell the probability of each level of stock, and this information might help forecasting storage costs. In particular, the overflow and the empty level probabilities, both critical to the design of the terminal capacity are determined. Most important is that although validated to the steady state the model can be especially useful for short and median-term decision making.

The second objective is to study a less general but very common situation in which the storage area is divided in two sectors, denominated main and secondary storage areas. The main sector would supposedly have the best responsiveness available, be covered by a complete set of high speed materials handling equipment, but be space constrained and having quite higher operating costs. The secondary storage sector would be able to absorb larger imbalances in the product flow, with very large or unlimited capacity, much lower operating costs, and it would be poorly materials-handling equipped. The transportation between the two storage sectors bears a volume dependent cost and access to and from the secondary area has to be through the main area. The modeled situation was suggested by the technical characteristics of the iron ore terminal examined but, in fact, it is a quite often found design restriction.

Previous work in these fields include the traditional contributions of Prabhu [4], Moran [2], Ghosal [1], and Hadley and Within [5]. With specific reference to the first objective the present research extends that of Pizzolato

and Videira [3].

The report is composed of five chapters. Chapters 2 and 4 introduce the proposed models to face respectively the first and the second objectives, while chapters 3 and 5 examine the application of each respective model to the data of the Québec Cartier iron ore terminal. Chapters 3 and 5 also contain a general evaluation of the results and the managerial possibilities introduced by the models.

II. THE SINGLE STORAGE MODEL

In order to describe the behavior of the terminal the states of the system will be associated to stock levels. For an integrated description, such a choice seems a natural one since the stock level is a common interface between the two interacting queues.

In the sequel, to simplify and consolidate the exposition, an iron ore exporting terminal will be assumed. In the terminal bulk arrivals are exclusively made by trains and dispatching is made by ships. The model proposed below is based on the following assumptions:

P1) Any train brings to the system a fixed amount of ore, denominated train unit, while ships carry integer random multiples of the train unit, according to a known distribution.

P2) The train unloading and the ship loading are instantaneous operations.

P3) Trains arrive according to a deterministic process and ships follow

a compound Poisson process, generated by a number of independent Poisson processes each one describing the arrivals of the corresponding ship type.

P4) Trains that arrive when the terminal is at or near capacity just discharge the necessary amount to complete the storage capacity and leave the system while arriving ships that do not find enough stock to satisfy their own demand empty the stock and leave the system.

Adequacy and consequences of these assumptions are examined at the end of this chapter but they are indeed extremely mild and might in many senses be trivially relaxed. As for assumption P3, which seems the most restrictive one, the proposed model shall only require minor adjustments if the two intervening arrival probability distributions are not those assumed but as long as they remain stochastically independent.

The following notation will be used where the system is observed at equally spaced instants t, t = 0,1,2,..., in which the time scale is the time interval T. These instants will be shown to be regeneration points of the Markov process focused.

E_t = State of the system at instant t, measured in number of train units of stock. $E_t \in \{0,1,2,\ldots,N\}$, where N is the storage capacity.

$P[E_t = n]$ = Probability of the system being in state n at instant t.

$\{a_j, j>0\}$ = Discrete probability density function of the number of train arrivals during a time interval T. According to assumption P3 and for a suitable choice of T there is an integer k such that $a_k = 1$, and $a_j = 0$ for $j \neq k$.

$\{b_j, j>0\}$ = Poisson probability density function $\{b_j = e^{-\lambda T}(\lambda T)^j/j!, j>0\}$ of the number of ships arrived during a time interval of

length T.

$B(z)$ = Generating function of the Poisson distribution $\{b_j,\ j\geq 0\}$, $B(z) = \sum_{j=0}^{\infty} b_j z^j = e^{-\lambda T+\lambda T z}$.

λ = Arrival rate of ships, where λ_r is the arrival rate of ships with capacity r, r = 1,2,....,R, and R is the maximum ship capacity and $\lambda = \sum_{r=1}^{R} \lambda_r$.

$\{c_r, 1\leq r\leq R\}$ = Discrete probability density function of ship capacity. As a consequence of the Poisson process, $c_r = \lambda_r/\lambda$.

$C(z)$ = Generating function of $\{c_r,\ 1\leq r\leq R\}$, $C(z) = \sum_{r=1}^{R} c_r z^r$.

$\{s_m, m\geq 0\}$ = Discrete probability density function of total ship capacity arrived during the time interval of length T, m=0,1,2....

$S(z)$ = Generating function of $\{s_m,\ m\geq 0\}$, $S(z) = \sum_{m=0}^{\infty} s_m z^m$.

2.1 DETERMINATION OF TOTAL SHIP CAPACITY

The initial step is the explicit determination of all terms of the probability density function $\{s_m,\ m\geq 0\}$ and this can be done in three alternative ways as shown next.

The first method makes use of generating functions and each term of S(z) is immediately seem as the sum of a random number of variables, leading to a branching process, for which the following probability variables are defined:

$C^{(k)}(z)$ = k-fold convolution of C(z) with itself, for k = 1,2,...., where $C^{(0)}(z) = 1$, and $C^{(k)}(z) = C(z)*C^{(k-1)}(z)$

$c_m^{(k)}$ = m-th power term of the k-fold convolution of C(z) with itself, k = 1,2,...., and m = k,k+1,....

Considering the above definitions and noting, for instance, that a ship

capacity of one requires the arrival of one ship and this ship shall be of size one; a ship capacity of two requires the arrival of either one ship of capacity two or two ships of combined capacity of two; a ship capacity of three requires the arrival of either one ship of capacity three or two ships of combined capacity of three, or three ships of combined capacity of three, and so on, it can be written:

$$s_0 = b_0$$
$$s_1 = b_1 c_1^{(1)}$$
$$s_2 = b_1 c_2^{(1)} + b_2 c_2^{(2)}$$
$$s_3 = b_1 c_3^{(1)} + b_2 c_3^{(2)} + b_3 c_3^{(3)}$$

or, in general:

$$s_m = b_1 c_m^{(1)} + b_2 c_m^{(2)} + \ldots + b_m c_m^{(m)} \quad \text{for } m = 1,2,\ldots$$

Multiplying corresponding terms by z^m , summing over all m, and performing standard derivation, the following expression is obtained:

$$S(z) = B(C(z)) \quad \text{or} \quad S(z) = e^{-\lambda T + \lambda T C(z)}$$

Substituting the expression for C(z) the values of $\{s_m\}$ can be recovered by successive derivation:

$$s_m = [d^m S(0)/dz^m]/m! \qquad \text{for } m = 1,2,\ldots$$

Instead of following this general procedure we choose for future reference to show the explicit solution for the first four terms. These are:

$$s_0 = b_0$$
$$s_1 = b_1 c_1$$
$$s_2 = b_1 c_2 + b_2 c_1^2$$
$$s_3 = b_1 c_3 + b_2 (2c_1 c_2) + b_3 c_1^3$$
$$s_4 = b_1 c_4 + b_2 (c_1 c_3 + c_2^2) + b_3 (3c_1^2 c_2) + b_4 c_1^4$$

The second method can be called a direct method and it makes use of direct probability arguments which enumerate the different streams of Poisson

processes in specific ways. Thus, for s_0, the probability of no ship during an interval of length T, it is simply given by the corresponding Poisson probability.

$$s_0 = b_0 = e^{-\lambda T}$$

A ship capacity of one train unit, s_1, corresponds to exactly one arrival of the Poisson stream type one and zero arrivals of the others types of ships. Consequently:

$$s_1 = P[1 \text{ of type } 1] * P[0 \text{ of type } i,\ i=2,3,\ldots,R]$$

$$= e^{-\lambda_1 T}(\lambda_1 T) \prod_{i=2}^{R} e^{-\lambda_1 T}$$

$$= e^{-\lambda T}(\lambda_1 T)$$

A ship capacity of two train units, s_2, can be obtained by either having one ship from the Poisson stream type two and zero ships of the others types, or by two ships of the Poisson stream type one and zero ships of the others. Consequently:

$$s_2 = P[1 \text{ of type } 2] * \prod_{i=1, i\neq 2}^{R} P[0 \text{ of type } i]$$

$$+ P[2 \text{ of type } 1] * \prod_{i=2}^{R} P[0 \text{ of type } i]$$

$$= e^{-\lambda T}[\lambda_2 T + (\lambda_1 T)^2/2!]$$

A total ship capacity of three trains, s_3, can be obtained by having either: exactly one ship of the Poisson stream type three and none of the others; one ship of the Poisson stream type one, one of the stream type two and none of the others, or, finally, by three ships of the stream type one and zero of the others. Consequently:

$$s_3 = P[1 \text{ of type } 3] * \prod_{i=1, i\neq 3}^{R} P[0 \text{ of type } i]$$

$$+ P[1 \text{ of type } 1] * P[1 \text{ of type } 2] * \prod_{i=3}^{R} P[0 \text{ of type } i]$$

$$+ P[3 \text{ of type } 1] * \prod_{i=2}^{R} P[0 \text{ of type } i]$$

$$= e^{-\lambda T}[\lambda_3 T + (\lambda_1 T)(\lambda_2 T) + (\lambda_1 T)^3/3!]$$

The next two terms are:

$$s_4 = e^{-\lambda T}[\lambda_4 T + (\lambda_3 T)(\lambda_1 T) + (\lambda_2 T)^2/2! + (\lambda_2 T)(\lambda_1 T)^2/2! + (\lambda_1 T)^4/4!]$$

$$s_5 = e^{-\lambda T}[\lambda_5 T + (\lambda_4 T)(\lambda_1 T) + (\lambda_3 T)(\lambda_2 T) + (\lambda_3 T)(\lambda_1 T)^2/2!)$$
$$+ (\lambda_2 T)(\lambda_1 T)^3/3! + (\lambda_1 T)(\lambda_2 T)^2/2!) + (\lambda_1 T)^5/5!]$$

The third method was suggested by W. Powell and could be denominated an algorithmic approach. The method is very easy to compute and it seems naturally preferable to the previous ones. As before, it works as long as the ship size probability density function, $\{c_r,\ 1<r<R\}$, is independent of the other random variables of the system, and its computational legitimacy can be proved by showing that it is equivalent to the results obtained by the first method above. It can be better understood by means of a table.

$\{s\} =$	0	1	2	3	4	...	R	R+1
b_0	1	0	0	0	0	...	0	0
b_1	0	c_1	c_2	c_3	c_4	...	c_R	0
b_2	0	0	$c_2^{(2)}$	$c_3^{(2)}$	$c_4^{(2)}$	...	$c_R^{(2)}$	$c_{R+1}^{(2)}$
b_3	0	0	0	$c_3^{(3)}$	$c_4^{(3)}$	...	$c_R^{(3)}$	$c_{R+1}^{(3)}$
b_4	0	0	0	0	$c_4^{(4)}$	...	$c_R^{(4)}$	$c_{R+1}^{(4)}$
.								
.								
.								
Total	s_0	s_1	s_2	s_3	s_4	...	s_R	s_{R+1}

The left side column gives the probabilities of an increasing number of arrivals during the time interval T, where $b_j = [(\lambda T)/j]b_{j-1}$, for $j = 1,2,\ldots$ and $b_0 = e^{-\lambda T}$.

In the body of the table lines 0 and 1 (corresponding to b_0 and b_1) are trivial. Line 2 is given by convolution of line 1 with itself, while the other lines are obtained by convolution of previous line with line 1. The total row line is simply the weighted sum of the column values, or:

$$s_0 = b_0$$

and $$s_m = b_m c_m + \sum_{i=2}^{\infty} b_i c_m^{(i)} \qquad \text{for } m = 1,2,\ldots$$

Once the values s_k obtained, and knowing the number of train arrivals k corresponding to $a_k = 1$, the system can be observed at each end of period. These instants are regeneration points of the Markov process because the last information is all that is needed to anticipate the next state position.

2.2 THE TRANSITION MATRIX

At a given instant t the system is in a particular state E_t. At a subsequent instant t+1 it will be in a different state as a result of two opposite movements: a k-states upward move with certainty, due to trains arrivals, and a random j-states downward move with corresponding probability s_j, due to ship arrivals. Recalling assumption P4 for the boundary values, the transition equations may be easily written.

$$P[E_{t+1} = 0] = \sum_{j=0}^{N} P[E_t = j] * \sum_{i=j}^{\infty} s_{k+i}$$

$$P[E_{t+1} = n] = \sum_{j=0}^{k1} P[E_t = n-j] * s_{k-j} + \sum_{j=1}^{N-n} P[E_{t-1} = n+j] * s_{k+j}$$

for $0 < n < k$ and $k1 = \min(n, k)$

$$P[E_{t+1} = N] = \sum_{j=0}^{k} P[E_t = N-j] * \sum_{i=0}^{k-j} s_i$$

The above transition equations are better seen as a stationary one step Markov transition matrix P.

$$P = \begin{array}{c|ccccc} & 0 & 1 & 2 & \cdots & N-1 \quad N \\ 0 & \sum_{i=0}^{\infty} s_{k+i} & s_{k-1} & s_{k-2} & \cdots & 0 \quad 0 \\ 1 & \sum_{i=1}^{\infty} s_{k+i} & s_k & s_{k-1} & \cdots & 0 \quad 0 \\ 2 & \sum_{i=2}^{\infty} s_{k+i} & s_{k+1} & s_k & \cdots & 0 \quad 0 \\ \vdots & & & & & \\ N-1 & \sum_{i=N-1}^{\infty} s_{k+i} & s_{k+N-2} & s_{k+N-3} & \cdots & s_k \quad \sum_{i=0}^{k-1} s_i \\ N & \sum_{i=N}^{\infty} s_{k+i} & s_{k+N-1} & s_{k+N-2} & \cdots & s_{k+1} \quad \sum_{i=0}^{k} s_i \end{array}$$

Denominating by X the column vector of steady state probabilities, i.e., $X = (x_0 , x_1 , \ldots , x_N)^T$, where $x_j = \lim P[E_t=j]$, the equilibrium of the system is represented by:

$$XP = X$$

or

$$X(P - I) = 0$$

Since all rows of the transition matrix P sum to one the above system

is linear dependent. As a consequence, its solution can be obtained by setting one of the x_j to an arbitrarily chosen value x'_j and solving the resulting system, no longer homogeneous. Once the solution obtained, a normalizing operation is applied to find the solution vector:

$$x_j = x'_j / \sum_{j=0}^{N} x'_j$$

From the solution vector other information of importance can be extracted: the probability of a ship leaving the harbour without satisfying its demand, or the expected number of unfulfilled demand in train units. Note that given the state of the system at instant t, for instance, $E_t = i$, then the expressions:

$$\sum_{j=1}^{\infty} s_{k+i+j} \qquad \text{and} \qquad \sum_{j=1}^{\infty} j s_{k+i+j}$$

give respectively the conditional probability of a ship leaving without satisfying its demand, and the expected number of unfulfilled train units. Consequently, the unconditional expected number train units not embarqued on ships at interval (t, t+1) is given by:

$$\sum_{i=0}^{N} x_i \sum_{j=1}^{\infty} j s_{k+i+j}$$

This expression shall be useful for designing the optimal N and for operating the terminal in such a way as to keep acceptably low the risk of not satisfying demand.

2.3 CRITIQUE OF THE ASSUMPTIONS

The effect of the simplifying assumptions ought to be examined and the following remarks seem pertinents:

1) With respect to assumption P1 which defines the stock unit as equivalent to one full train of mineral, this hypothesis may be easily relaxed and adapted to each specific case. Suppose. for instance, that train sizes are not fixed but they vary according to some probability law. In this case, the adequate stock unit might be the rail car, or a fixed number of rail cars, rather than a whole train. As a consequence, the states E_t should be redefined as the probability of n rail car units, or a n-multiple of the fixed number of rail cars in storage at time t. On the other hand, the ship size probability density function (c_r, $1 \leq r \leq R$) should also be redefined as the number of equivalent rail cars transported by a ship. Such extension is straightforward but not presented since it keeps great similarity with the procedures adopted before, with considerable increase in the number of states of the system.

2) Considering assumption P2, about the instantaneous loading and unloading operations, it should when applied to modern terminals with extremely fast operations bring no significative discrepancy over the model predictions. On the other hand, for terminals with slow operations, the model shall underestimate the real stock in a volume equivalent to one half the expected capacity of one ship times the number of simultaneous ship loading berths times their expected occupancy rate. For short term forecasts of the stock behaviour, however, the above effect is meaningless because the initial stock

level shall already reflect the slow mode in which the terminal operates.

3) The crucial aspect behind assumption P3 is the stochastic independence between train and ship arrivals. Although several forms of dependence might be incorporated on the transition matrix, its no satisfaction could invalidate the model as proposed above, since stock levels shall be inferior to model predictions. An extreme case of stochastic dependence is the one of perfect coordination between trains and ships, when stocks in the terminal would be almost null. Thus, maintained the stochastic independence the one step transition matrix may quite accurately accomodate any sort of probability density function for both trains and ships.

4) Although assumption P4 seems at first sight quite restrictive, not only its interpretation and effects might be easily explained but also it can be relaxed at will. Obviously, for intermediate stock levels the assumption has no significant effect, but distortions between model and reality due to the truncation made are likely to occur near the extreme values. Therefore, the suggestion is to artificially extend the stock boundaries, 0 to N, to, say, $-N_1$ to N_2, where $-N_1 < 0$ and $N_2 > N$ where N_1 and N_2 reflect real operating policies. So, N_1 can represent the maximum delay ships agree to wait for completing the cargo beyond which they leave or simply cancel their arrival. Analogously, to avoid stock overflow, N_2 can represent the maximum number of trains whose discharging can be postponed without disturbing the railroad and mining operations.

2.4 THE SHORT-TERM DECISION PROCESS

One significative importance of the model is related to the short term. Suppose $X^{(0)}$ and $X^{(n)}$ are defined as:

$X^{(0)} = [x_0^{(0)}, x_1^{(0)}, \ldots, x_N^{(0)}]$ = initial state vector

$X^{(n)} = [x_0^{(n)}, x_1^{(n)}, \ldots, x_N^{(n)}]$ = n-iteration state vector

$X = \lim X^{(n)}$ = stationary probability vector

The following are standard relations:

$$X^{(n)} = X^{(n-1)}P \qquad \text{for } n = 1,2,\ldots$$

and

$$X^{n} = X^{(0)}P^{n} \qquad \text{for } n = 1,2,\ldots$$

The last expression is particularly important since once the present state of the system is known its probability distribution n periods ahead can be calculated. Exactly here lies the relevance of the model to the short-term decision process as outlined by a number of hypothetical situations as discussed below.

a) Suppose the railroad will be closed for repairs for a known number of days, say, L days. The probability of running out of stock during the shutoff period can be easily determined. It is necessary to recalculate the transition matrix P for the new situation in which there is no arrival, and this can be done by fixing k=0 in the transition matrix P given before, changing it to P', and applying the expression:

$$X^{(L)} = X^{(0)}P'^{L}$$

Certainly, to reduce the risk of no stock, $X^{(0)}$ can be seen, in the

expresion above, as a decision variable, and the problem could be put in the following terms: At what minimim level $X^{(0)}$ should the stock be before the service interruption occurs so as to keep the zero stock probability satisfactorily low?

b) Suppose the railroad may get closed in some seasons of the year by an action of nature such as rain storms in tropical and subtropical areas or snow storms in cold areas. Through historical data, topographical, logistics, and engineering considerations, or, simply, by opinion analysis, it is possible to devise the construction of a probability distribution associated to each possible number of days required to restore the normal trafic, if the disaster happens. If L represents the most unfavorable situation, the same arguments given above can be applied and a minimum goal stock $X^{(0)}$ at the beginning of the unfavorable season should be pursued. This is a simple method to face the problem but the rationale can be carried much further by considering the prospective costs of being out of stock against the cost of anticipating larger amounts of stock to the terminal and handling them.

c) Suppose the ship capacity probability density function (c_r) changes for a number of reasons: seasonal effects, new customers, new kinds of ships, or a shift in the arrival rate. If these changes are known in advance this can be reflected by a different transition matrix P' and its consequences evaluated in terms of either $X^{(n)}$ or the stationary vector X. This analysis may suggest, for instance, changes in the train arrival rate and, consequently, in the mining operations.

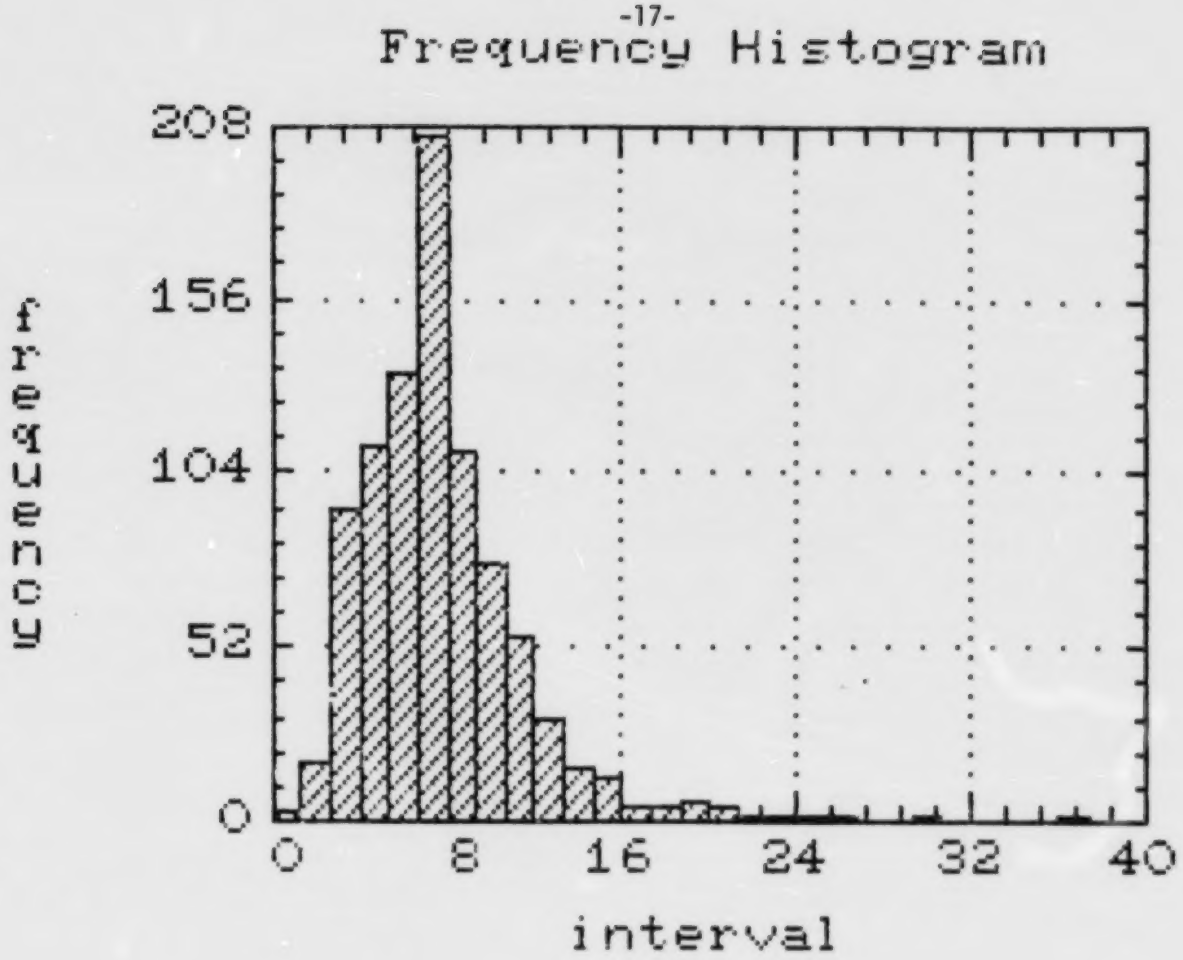
III. A REAL APPLICATION

The model proposed before was applied to the 1986 data of the Quebec Cartier iron ore terminal. This chapter discusses the problems related to this real world implementation such as data collection, testing of assumptions, evaluation of results, etc.

The time period chosen was the day, or T = 24 hours, and the subsequent presentation will clarify the reasons for this choice. The discussion that follows is divided in four parts. First, data relative to trains are reviewed. Next, data relative to the ships. Then, the solution of the system is presented and this constitutes the expected probabilities. Finally, a tentative evaluation and analysis of the results is made.

3.1 TRAINS

Ten months of information were available producing a total of 897 trains with an average interarrival time of 7.927 hours, which is equivalent to three trains a day, and a sample variance of 25.72 hours^2. The large variance was mainly due to two observations resulting from two four-days periods without any train. Removing these two trains from the sample data, the resulting frequency histogram can be seen in Figure 3.1 or as a frequency distribution in Table 3.1. Both displays show that the deterministic distribution seems a good descriptor of the number of trains in a single day, since more than half of the trains are on a single class, between 4.90 to 9.80 hours, and practically all other observations belong to the next lower and upper classes. An alternative and apparently more desirable way to present the information would be the



ENTER THE NAME OF THE VARIABLE CONTAINING YOUR DATA: interi
NUMBER OF OBSERVATIONS = 894 (1 MISSING VALUES EXCLUDED)
SAMPLE AVERAGE = 7.78543
SAMPLE VARIANCE = 15.2
SAMPLE STANDARD DEVIATION = 3.89872

MINIMUM VALUE = 0.25 MAXIMUM = 41.17 RANGE = 40.92
LOWER AND UPPER QUARTILES = 5.33 9.25
INTERQUARTILE RANGE = 3.92
MEDIAN = 7.25

COEFF. OF SKEWNESS = 2.31565 STANDARDIZED VALUE = 28.2662
COEFF. OF KURTOSIS = 14.6395 STANDARDIZED VALUE = 71.0392
Press ENTER to continue.

FIGURE 3.1: Train Interarrival Time in Hours

frequency distribution of number of trains in a single day. This information, however, was not directly available but it would certainly be dependent on the cut time as, for instance, a four-trains day could well be followed by a two-trains day.

FREQUENCY TABULATION

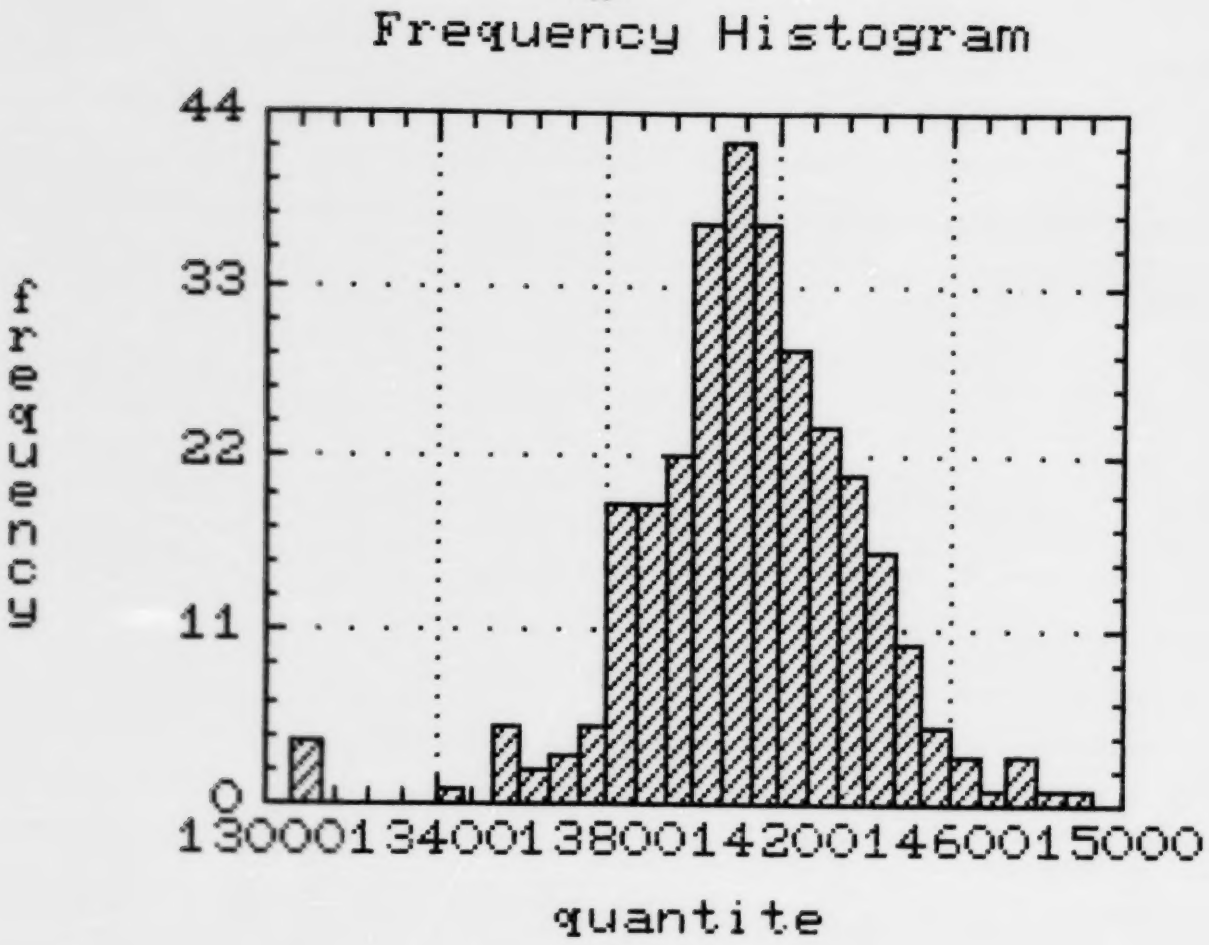
CLASS	LOWER LIMIT	UPPER LIMIT	MIDPOINT	FREQUENCY	RELATIVE FREQUENCY	CUMULATIVE FREQUENCY	CUM. REL. FREQUENCY
AT OR BELOW		.00		0	.00000	0	.000
1	.00	4.90	2.45	181	.20246	181	.202
2	4.90	9.80	7.35	531	.59396	712	.796
3	9.80	14.70	12.25	142	.15884	854	.955
4	14.70	19.60	17.15	24	.02685	878	.982
5	19.60	24.50	22.05	11	.01230	889	.994
6	24.50	29.40	26.95	2	.00224	891	.997
7	29.40	34.30	31.85	1	.00112	892	.998
8	34.30	39.20	36.75	1	.00112	893	.999
9	39.20	44.10	41.65	1	.00112	894	1.000
10	44.10	49.00	46.55	0	.00000	894	1.000
ABOVE	49.00			0	.00000	894	1.000

MEAN = 7.7854 STANDARD DEVIATION = 3.8987 MEDIAN = 7.25

TABLE 3.1: Train Interval Time in Hours

Although the present application is assuming a stable k = 3 trains a day, which accurately defines the yearly average, the railroad is clearly able to sustain more than three trains a day. In fact, the arrival data suggests that to attain the mentioned average, the railroad has to compensate during the summer for the periods it is closed due to maintenance or, especially, during the winter time when the production process at the mine level has to overcome a number of problems which, if do not force its closure at all, tend to slow its operations down considerably.

Regarding the volume transported by each train, Figure 3.2 shows its frequency distribution. Out of 315 observations, the average was 14,116 ton,



ENTER (1) NAME OF THE VARIABLE CONTAINING YOUR DATA: quantite
NUMBER OF OBSERVATIONS = 315 (552 MISSING VALUES EXCLUDED)
SAMPLE AVERAGE = 14116
SAMPLE VARIANCE = 256465
SAMPLE STANDARD DEVIATION = 506.36

MINIMUM VALUE = 9304 MAXIMUM = 19338 RANGE = 9934
LOWER AND UPPER QUARTILES = 13990 14283
INTERQUARTILE RANGE = 293
MEDIAN = 14126

COEFF. OF SKEWNESS = 0.35764 STANDARDIZED VALUE = 2.5914
COEFF. OF KURTOSIS = 62.917 STANDARDIZED VALUE = 217.18
PRESS RETURN to continue

FIGURE 3.2: Tonnage Carried by Trains

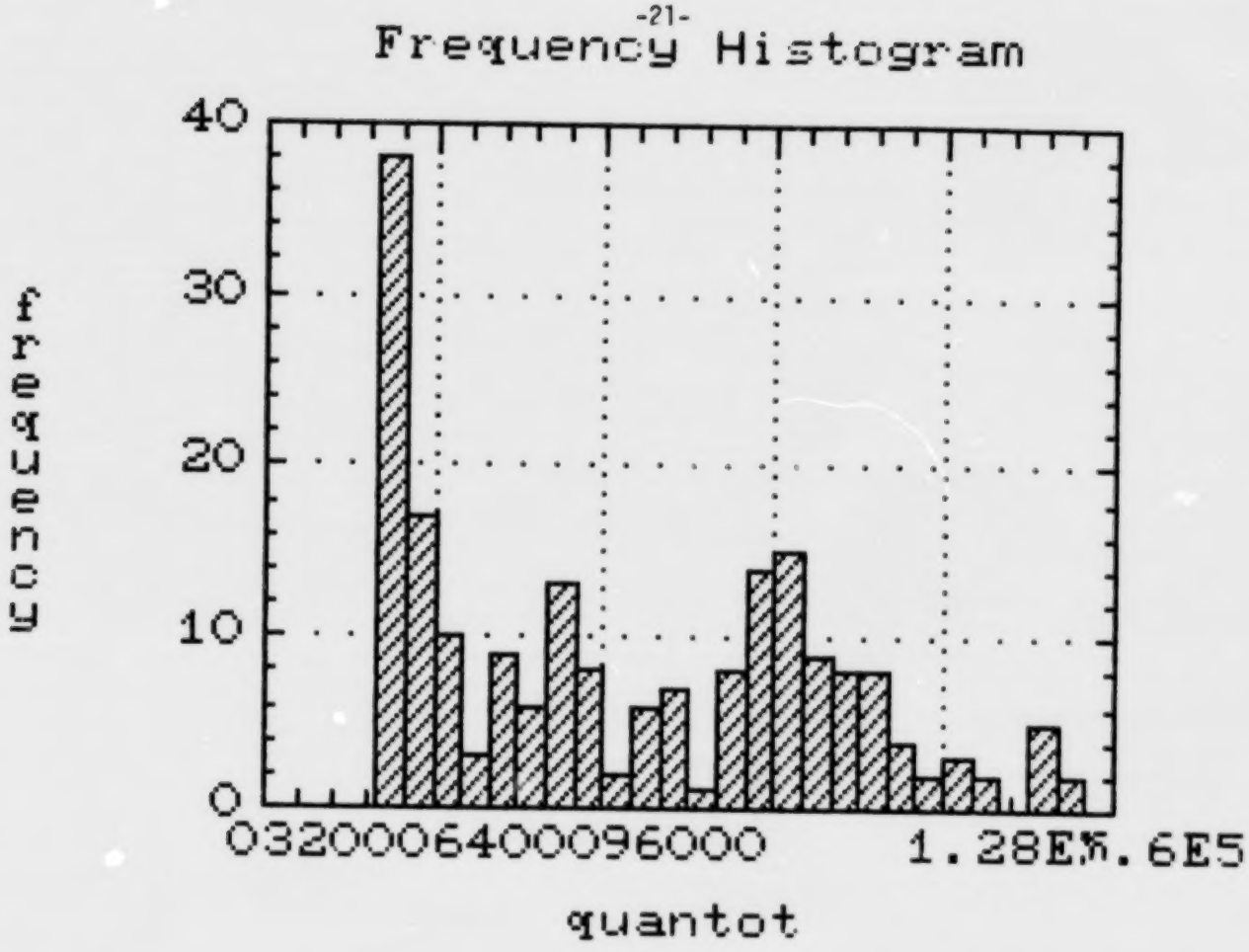
with a standard deviation of 506.36 ton and, therefore, the volume per train can be considered very stable. In addition, if three observations are eliminated, all others are on the range 13,400 to 15,000 ton, with almost all of them belonging to the 13,800 to 14,600 range. As a consequence, the train unit was taken to be 14,000 ton.

3.2 SHIPS

During the year, 200 ships have docked at the terminal. The attached frequency histogram (Figure 3.3) for the entire year shows that ship sizes are broadly dispersed from a smallest size of 21,395 ton to a maximum of 154,616 ton. The referred histogram suggests that the grouping of ships according to their sizes is quite justifiable. Therefore, ships have been classified in five classes in multiples of 14,000 ton. Table 3.2 shows a total of 200 observations grouped in class intervals given in tons, and in trains, and also the class mid points in tons and in trains, as well as the class frequency.

Class interval in tons	Class interval in trains	Mid point tons	Mid point trains	Frequency
14000- 42000	1-3	28000	2	66
42000- 70000	3-5	56000	4	40
70000- 98000	5-7	84000	6	42
98000-126000	7-9	112000	8	38
126000-above	> 9	140000	10	14

TABLE 3.2: Frequency Distribution of Tonnage Carried by Ships



ENTER THE NAME OF THE VARIABLE CONTAINING YOUR DATA: quantot
NUMBER OF OBSERVATIONS = 200 (0 MISSING VALUES EXCLUDED)
SAMPLE AVERAGE = 60425.8
SAMPLE VARIANCE = 1.364886E9
SAMPLE STANDARD DEVIATION = 36914

MINIMUM VALUE = 21395 MAXIMUM = 154616 RANGE = 133221
LOWER AND UPPER QUARTILES = 30539 98948
INTERQUARTILE RANGE = 68409
MEDIAN = 50964

COEFF. OF SKEWNESS = 0.352471 STANDARDIZED VALUE = 2.03499
COEFF. OF KURTOSIS = -1.9347 STANDARDIZED VALUE = -3.07469
Press ENTER to continue.

FIGURE 3.3: Frequency Histogram of Tonnage Carried by Ships

Table 3.3 is more descriptive. It gives the number of ships arrived in each month classified according to its size. Moreover, the table shows for each ship class the average monthly arrival rate, the variance of arrivals and the χ^2-test, assuming a Poisson distribution.

The test indicates that except for the 5 to 7 train size class, there is no reason to reject the Poisson hypothesis but the evaluation of the Table requires several comments. For instance, the addition of the five class variances largely exceeds the total variance - 23.9 against 12.6. Thus, while two classes have larger than Poisson variances the total number of ships has a variance consistent with the Poisson hypothesis. This fact points out the existence of a negative correlation among classes, which indicates that some dependence among classes seems to exist. In other words, a reduction in the occurrences of one ship size class may activate others so that the total number of ships remain a well behaved Poisson. As a consequence, the convolution used to estimate the probability of total ship capacity arrived during a certain period may overestimate it.

Another behaviour not properly detected by the χ^2-test but noticed in the practical operations, and confirmed by the data, is the clear traffic reduction for small ships in January and February and its augmentation during the summer months. The reason is known: almost all small ships go to the Great Lakes and this traffic is interrupted by the occurrence of ice that block the way. This interruption seems to be compensated during the summer months. In any case, the large variance with respect to the mean indicates for the smaller ships some seasonal trend.

Pursuing further this analysis, it is possible to say that as a consequence of the above remarks, the model would underestimate ship capacity during the summer months and would overestimate it during two of the winter months.

Ships	Month														χ^2 Test		
	1	2	3	4	5	6	7	8	9	10	11	12	$\hat{\lambda}$	s^2	$\chi^2_{cal.}$	$\chi^2_{.01}$	$\chi^2_{.05}$
1-3 trains	2	1	4	4	13	11	7	4	3	6	5	6	5.5	12.3	1.57	13.28	9.49
3-5 trains	6	3	6	3	3	1	1	5	4	1	3	4	3.3	3.2	3.77	7.82	7.82
5-7 trains	1	6	3	7	5	1	3	0	5	1	5	5	3.5	5.4	11.76	13.28	9.49
7-9 trains	5	4	1	5	2	1	4	3	4	1	5	3	3.16	2.5	1.92	11.35	7.82
9 trains +	2	0	0	1	1	1	1	2	1	2	2	1	1.16	.5	1.36	6.64	3.84
Total	16	14	14	20	24	15	16	14	17	11	20	19	16.6	12.6	3.66	11.35	9.82

Total no. of ships: 200

Table 3.3: Distribution of ships arrivals according to size.

However, there is also an opposite phenomenon from the supply side, as noted before, which is the production slow down at the mine level during the winter months. Consequently, the combined effect is softened out.

For determining the total ship capacity arrived during each time period, (s_m, m>0), the first one of the methods discussed on section 2.1 was employed. Results are indicated in Table 3.4 in which the probabilities were truncated at the ship capacity s = 20, and subsequently normalized to add to one.

s	Probability	s	Probability
0	.581264	12	.016975
2	.105105	14	.011848
4	.073202	16	.007695
6	.078975	18	.004132
8	.077167	20	.001595
10	.042042		

TABLE 3.4: Ship Capacity Probability

3.3 SOLUTION OF THE SYSTEM

The solution of the system is shown in Figures 3.4 and 3.5 but before attempting any interpretation it is necessary to notice that the results are certainly affected by the fact that during the year under study, 1986, there has been a significative accumulation of stock at the terminal. The above statement can be supported from both the expected probabilities as well as from the actual stock data.

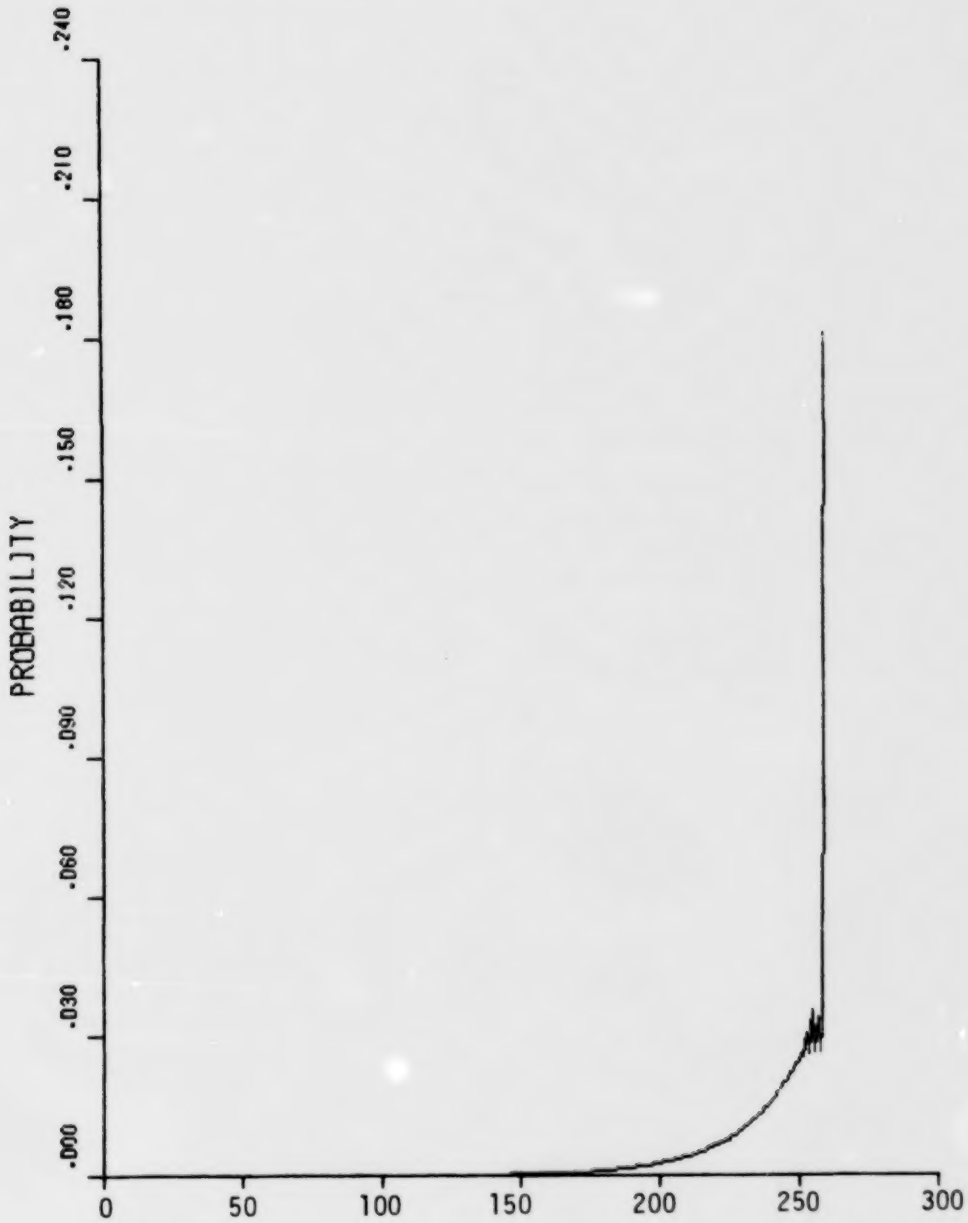


FIGURE 3.4: Steady State Probabilities

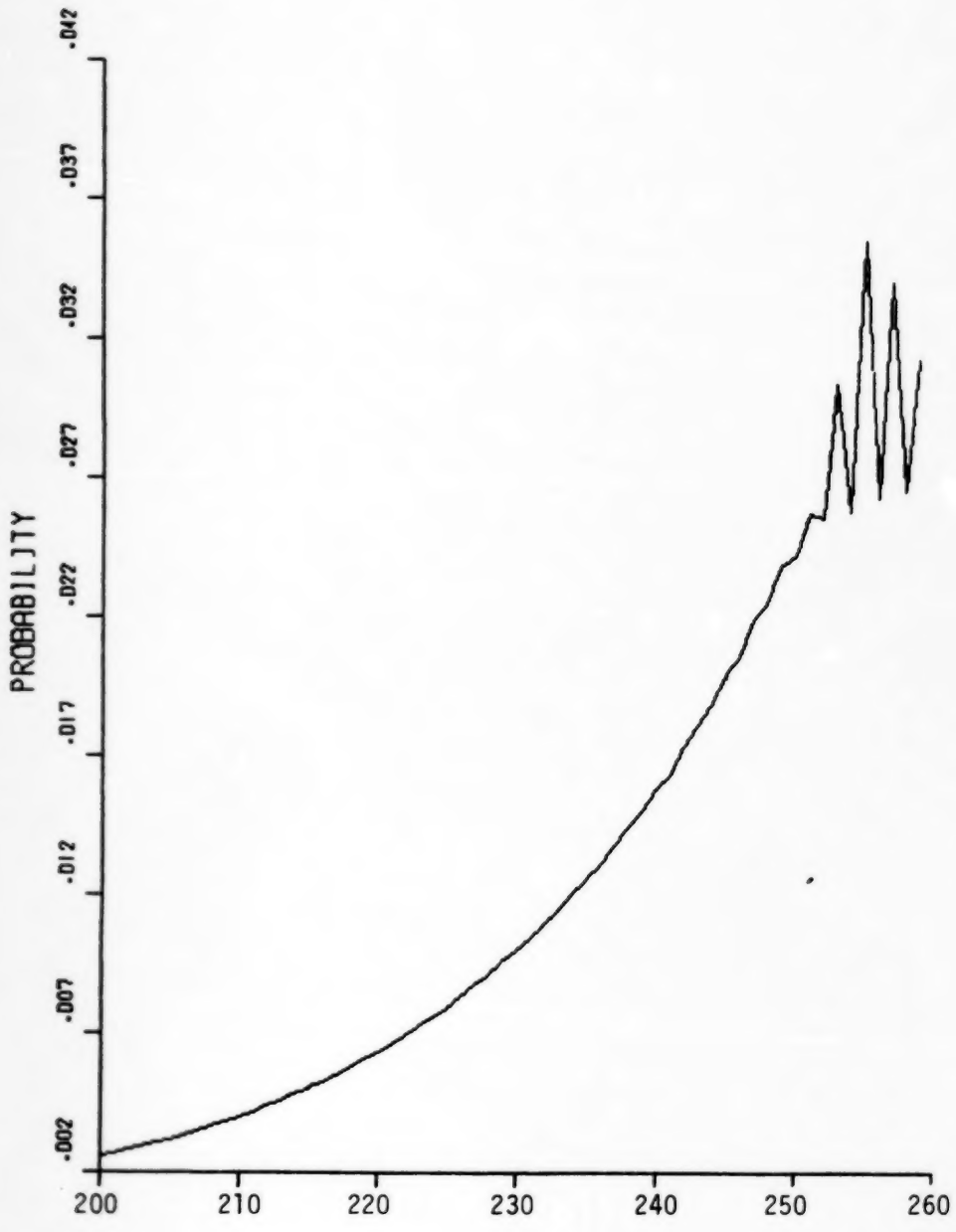


Figure 3.5 System Solution in the 200-259 Range

Consider first the expected probabilities. The expected value of the total ship capacity arrived during one day can be calculated from the probability Table 3.4 and it gives the expected value E[s] = 2.61 trains a day embarked on ships. Comparing this number with k=3 trains that arrive every day there is a daily surplus of .39 trains which along the year produce an excess of:

$$(3 - E[s]) * 360 = 119.34 \text{ trains}$$

which represent about one half of the capacity of the terminal.

Relative to the actual stock data the available records are very much undocumented but according to the best recollection the total stock at the terminal was:

- end January: 1,067,000 ton or 76.2 trains
- end September: 3,080,000 ton or 220 trains

The high stock level at the start of the winter period seems to confirm some previous remarks about seasonalities, i.e., there is a production slow down with exports maintained despite the reduction in the traffic of smaller ships. Consequently, the iron ore inventories at the terminal seem to play the fundamental role of smoothing out the production system. Unfortunately, the lack of reliable files do not allow the practical testing of the above statements.

Returning now to Figures 3.4 and 3.5 their understanding became easier. Thus, Figure 3.4 gives the steady state probabilities which according to the previous remarks tend consistently to the upper side of the state range. Therefore, for states below 150 all probabilities are near zero while for states approaching the upper limit they show an increasing trend and for the limit 260 there is a high concentration of probability.

Figure 3.5 shows a partial picture of the solution, i.e., the 200-259 range. The saw kind of graph at the extreme right is consistent with the data given by Table 3.4 above which says that ship sizes are given by even multiples of trains.

3.4 EVALUATION OF THE RESULTS

Denoting by Z_t the random variable which describes the variation of the state of the system from time t-1 to t:

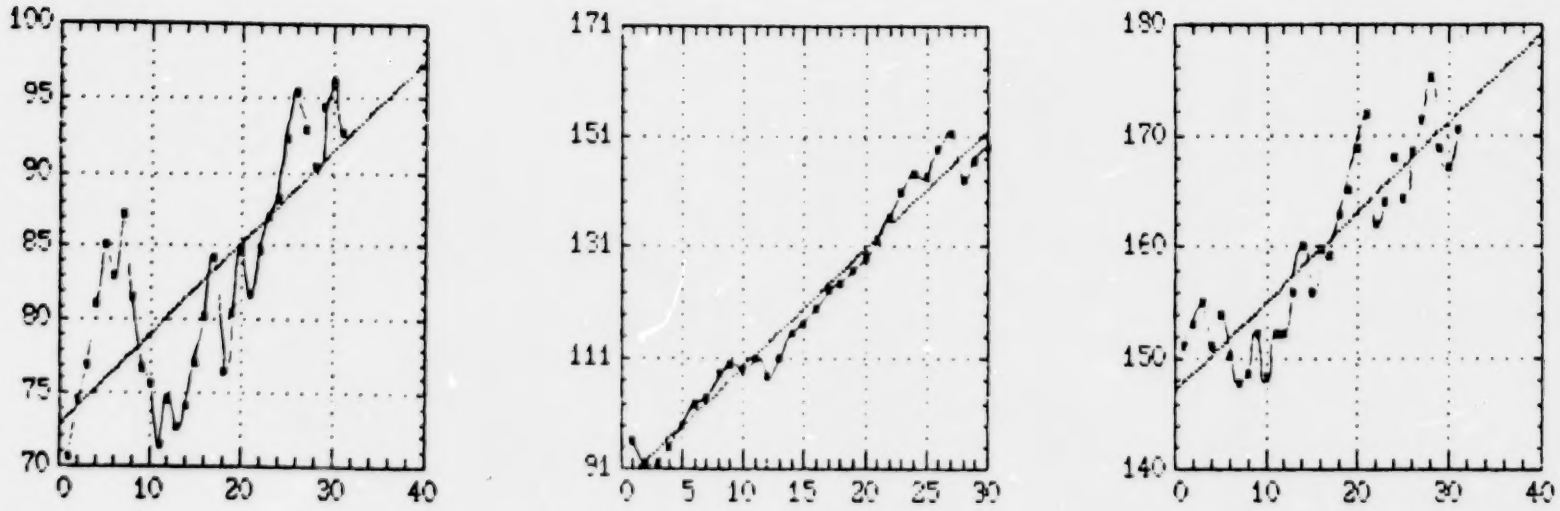
$$Z_t = E_t - E_{t-1}$$

In the steady state, the variable Z_t is independent of the time and the subscript t can be dropped. Therefore, using the data defined previously:

$$E[Z] = \sum_{j=0}^{20} (3 - j)\, s_j = .29$$

$$VAR[Z] = \sum_{j=0}^{20} (3 - j - E[Z])^2\, s_j = 15.46$$

Figure 3.6 shows for the months of May, June, and July the behaviour of the actual stock at the terminal against the trend determined by linear regression. The resulting variances for the months of May, June, and July are respectively 24.79, 10.16, and 13.57. The referred figure suggests the fact that besides the trend that the terminal is being slowly filled out, there are random fluctuations. If the short term line determined by linear regression were stable and equivalent to the long range trend given by E[Z] then the random fluctuations of the actual stock would be close to VAR[Z]. In fact, under the limitations of extrapolating the steady state from a single month



MAY JUNE JULY

FIGURE 3.6: Actual and Predicted Stock in Selected Months

view, June and July would fit in the model while the variance of May is higher than the one predicted.

3.5 CONCLUSIONS

The design of any terminal is supported by at least two queueing models: one describing the train arrivals and defining the technical characteristics of the discharging equipments. The second queueing finds the tradeoff between number of berths and the ships waiting time cost, as a function of ship arrival rate and service time.

The model proposed here adds another desirable element to a terminal project, i.e., its surface or inventory capacity. At first, admitting independence between arrival of trains and ships the model predicts the probability of different levels of stock and these would represent upper bounds. Lower levels of stock could be attained in case the operation of the system allows at reasonable costs for some degree of control to adjust the mining and railroad operations to the arrival of ships.

Second, the model may be very useful for short term decisions. So, if the railroad is to be closed for repairs for a planned number of days the level of stock prior to its closure can be determined so that risks associated to the lack of stock can stay reasonably low.

A third and similar situation is associated to the closure of the railroad by accidents or by actions of nature in specific periods of the year. By an educated judgement of the length of time required to put it back to normal operation it is possible to determine the level of stocks to carry before the unfavorable season.

Fourth, if there is an increase in the ship traffic one solution is certainly to increase the surface of the terminal. Another solution in case the surface is fixed is to impose a closer control of the train arrival rate which, in turn, shall also require a closer monitoring of the mining operations.

IV. THE DOUBLE-STORAGE MODEL

The situation contemplated now is one in which the storage area is divided in two sectors: a main and a secondary area. In practice, such construction feature is found quite often for two reasons. One, is the fact that many terminals are developed in harbour areas which are already congested and space constrained so that the area available is either very limited or too expensive making necessary the development of an additional nearby area of lesser cost to accommodate massive volumes of stock. Secondly, high capacity recuperation equipments used to collect the mineral off the ground are particularly expensive and thus it is often preferable to make it only cover a restricted surface. When the stock overflows this limited area the use of tractors, reduced speed conveyors, and other lower cost auxiliary means is required to bring the mineral within the reach of the recuperation equipment. In one or the other situation, the question of basic interest is the determination of the size of both the main and the secondary areas so as to reduce capital and operation costs.

The main storage sector is responsible for all the interface with unloading trains and loading ships but, by force of design, it holds a maximum stock level. In addition, since the ships are loaded from the mineral stock laying in the main area, it has also to operate at a minimum stock level to avoid ship delays. To achieve these two operating bounds, there is a secondary storage sector which acts in two ways: it receives the incoming mineral when the capacity at the main storage is exceeded, and it resupplies the main storage when the stock is below the minimum operating level. Physically, the communication between the two sectors is made by conveyors and recuperation equipments. Since this internal movement is typically expensive and does not

add anything to the production process a clear objective is to reduce this dead movement. Certainly, the knowledge of the conveyor working frequency shall help the design of the terminal facilities and, especially, the determination of the optimum trade-off between investment and operating costs.

In other words, the system appears similar to the popular (s,S) policy used in inventory control but it is not the same and to avoid misunderstanding it will be referred further as the (m,M) policy. In contrast to the (s,S) case, in the system studied here the products are resupplied permanently by the train arrivals and are demanded according to ship arrivals resulting in a random movement of the net stock level. However, if the stock level goes beyond M, conveyors transfer all the excess to the secondary storage; if the stock gets below m conveyors resupply, in addition to the trains, the main storage area until the minimum level is attained or the secondary stock depleted.

To solve the problem described above three models are proposed below. The first requires a two dimensional state space in which the state of the system is defined according to the stock level at each of the two sectors. It is certainly the most descriptive of them, but its clear disadvantage is the possible excessive number of state combinations and computing requirements. The second approach simplifies the problem to focus only on the main storage level and its relationship with the secondary storage making use of the concept of reflecting barriers used here as a proxy to finding the conveyor working frequency. The third and last model extends the second one by admitting the realistic supposition that the only effective boundary is M while the lower limit m is a flexible bound that exists to make sure ships are loaded without delay.

The models presented next are based on the same assumptions stated before and the notation remains that stated of section II. In addition, the

conveyor speed needs to be defined and limited additional notation will be introduced.

4.1 THE TWO-DIMENSIONAL STATE MODEL

In this first double storage model the system description requires the identification of two elements, i.e., the stock level at each of the two storage sectors. When the stock at the main storage area reaches M any incoming product goes to the secondary storage while when the stock level is below m products from the secondary storage are transferred to the main sector. With this structure in mind, one additional notation and one additional assumption are necessary. These are:

$E_t(n,h)$ = State of the system in which there are n train units in main storage and h train units in secondary storage at instant t. $0 \leq n \leq M$, $0 \leq h < \infty$.

P5) The stock at the main sector fluctuates between an upper and a lower level assisted by instantaneous conveyors. The secondary storage holds any stock that exceeds the upper level and, when the stock is below the lower level, it resupplies the main storage up to the minimum or until it is depleted.

With the elements above the model follows. However, before writing the transition equations the reader shall appreciate seeing the state equations which are considerably simpler. For this, let the following variables be defined, all measured in number of train units:

X_t = stock level in main storage at instant t.

Y_t = stock level in secondary storage at instant t.

U_t = amount of stock moved from the main to secondary storage during the time interval (t, t+1).

V_t = amount of stock moved from the secondary to main storage during the time interval (t, t+1).

D_t = satisfied demand during the time interval (t, t+1).

S_t = demand generated by ships during the time interval (t, t+1)

Using the notation $[X]^+ = \max(0, X)$, and recalling that k is the fixed number of trains during the time interval (t,t+1), it follows that:

$$X_{t+1} = [X_t + k + V_t - s_m - U_t]^+$$

$$Y_{t+1} = [Y_t + U_t - V_t]^+$$

$$V_{t+1} = \min(Y_t, [s_m - X_t - k + m]^+)$$

$$D_{t+1} = \min(s_m, X_t + k + Y_t)$$

The transition equations now follow:

$$P[E_{t+1}(M, h)] = \sum_{j=0}^{k1} P[E_t(M, h-j)]s_{k-j} + \sum_{j=1}^{k} \sum_{i=j}^{k} P[E_t(M-j, h+j-i)]s_{k-i}$$

for $0 < h < \infty$ and $k1 = \min(h, k)$

$$P[E_{t+1}(n, h)] = \sum_{j=1}^{M-n} P[E_t(n+j, h)]s_{k+j} + \sum_{j=0}^{k1} P[E_t(n-j, h)]s_{k-j}$$

for $m < n < M$: $0 < h < \infty$ and $k1 = \min(k, n-m)$

$$P[E_{t+1}(m,h)] = \sum_{j=0}^{M-m} \sum_{i=0}^{\infty} P[E_t(m+j,h+i)]s_{k+j+i}$$

for $0 < h < \infty$

$$P[E_{t+1}(M, 0)] = \sum_{j=0}^{k1} P[E_t(M-j, 0)]s_{k-j}$$

for $k1 = \min(M, k)$

$$P[E_{t+1}(n, 0)] = \sum_{j=0}^{k1} P[E_t(n-j, 0)]s_{k-j} + \sum_{j=1}^{M-n} P[E_t(n+j, 0)]s_{k+j}$$

for $m < n < M$ and $k1 = \min(n, k)$

$$P[E_{t+1}(m,0)] = \sum_{j=0}^{k1} P[E_t(n-j,0)]s_{k-j} + \sum_{j=1}^{M-n} \sum_{h=0}^{\infty} P[E_t(n+j,h)]s_{k+j+h}$$

for $0 < n \leq m$ and $k1 = \min(n,k)$

$$P[E_{t+1}(0, 0)] = \sum_{j=0}^{M} P[E_t(j, 0)] \sum_{i=j}^{\infty} s_{k+i} + \sum_{j=m}^{M} \sum_{h=0}^{\infty} P[E_t(j,h)] \sum_{i=j+h}^{\infty} s_{k+i}$$

The advantage of the present model lays in its similarity with the one on section II, but beyond this it provides two types of information critical for both the design process and the management of the short term. Similarly to the previous model it gives a general probability description of the whole system. On the other hand, it also indicates for the period (t, t+1) the frequency of utilization of the conveyor as well as the probability of not satisfying the demand. These are:

$\sum_{h=0}^{\infty} \sum_{j=0}^{k-1} P[E_t(M-j, h)] \sum_{i=j+1}^{k} s_{k-i}$ = probability of using the conveyor away sense

$\sum_{h=1}^{\infty} \sum_{i=0}^{M-m} P[E_t(m+i,h)] (\sum_{j=1}^{h} s_{k+j+i} + \sum_{j=h+1}^{\infty} s_{k+j+i})$ = probability of using in the towards sense

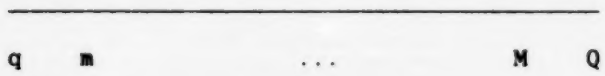
$\sum_{h=1}^{\infty} \sum_{h=0}^{M-m} P[E_t(n,h)] \sum_{j=n+h+1}^{\infty} s_{k+j}$ = Probability of not satisfying demand

The obvious disadvantage of this model is the excessive number of states generated by the solution process. Thus, if H is the maximum number of values h can assume the transition matrix will be of dimension (M*H, M*H).

Note that assumption P5 is not unrealistic but it shall not be difficult, although tedious, to relax it and to suppose a low speed conveyor. In any case, the next model will hold assumption P5 while the third model will relax it.

4.2 THE TWO-REFLECTING STATES MODEL

In contrast to the previous model, in the present one only the stock level in the main storage is controlled. As a consequence, the stock in the secondary storage is assumed endless and also of unlimited capacity. The relationship between both areas is supposed to take place whenever the stock reaches a virtual state Q which is one stock unit beyond M, or when it reaches a virtual state q which is one stock unit below m, as shown in the figure:



The two virtual states Q and q are called reflecting states because with probability one they return the system to respectively states M and m. The physical interpretation of states Q and q is that by the action of the conveyors the system is returned to its operating bounds, M and m. In other words, state Q means that products are removed out of the main storage area, while state q means that products are transferred to the main storage area. The stationary probabilities $P[E_t = Q]$ and $P[E_t = q]$ represent the probability of

having the conveyor working in corresponding directions although, in general, it does not indicate the amounts transferred.

For the design process the problem can certainly be put in other terms: which pair of values (m, M) to choose, so that the utilization of the conveyors stays reasonably limited? It may be correctly argued that in face of assumption P5 the lower level m can be made arbitrarily low. Therefore, the value m has to be set exogenously and the model shall help determining M.

Given this preliminary setting, using the previous notation and assumptions, and noting that assumption P5 is fully used, the transition equations are:

$$P[E_{t+1} = Q] = \sum_{j=0}^{k-1} P[E_t = M-j] \sum_{i=0}^{k-j-1} s_i + P[E_t = Q] \sum_{i=0}^{k-1} s_i$$

$$P[E_{t+1} = M] = \sum_{j=0}^{k} P[E_t = M-j]\, s_{k-j} + P[E_t = Q]\, s_k$$

$$P[E_{t+1} = n] = \sum_{j=0}^{M-m} P[E_t = n+j] s_{k+j} + \sum_{j=1}^{k1} P[E_t = n-j]\, s_{k-j}$$

$$+ P[E_t = Q]\, s_{k+M-n} + P[E_t = q]\, s_{m+k-n}$$

for $m < n < M$ and $k1 = \min(n-m, k)$

$$P[E_{t+1} = m] = \sum_{j=0}^{M-m} P[E_t = m+j]\, s_{k+j} + P[E_t = Q]\, s_{k+M-m} + P[E_t = q]\, s_k$$

$$P[E_{t+1} = q] = \sum_{j=0}^{M-m} P[E_t = m+j] \sum_{i=j+1}^{\infty} s_{k+i} + P[E_t = Q] \sum_{i=M-m+1}^{\infty} s_{k+i}$$

$$+ P[E_t = q] \sum_{i=1}^{\infty} s_{k+i}$$

The corresponding transition matrix, supposing $k < M-m$, is given below. The equality between lines q and m, and M and Q, should help the reader understand the assertions made above.

$$
P = \begin{array}{c|cccccccc}
 & q & m & m+1 & m+2 & \cdots & M-1 & M & Q \\
q & \sum_{i=1}^{\infty} s_{k+i} & s_k & s_{k-1} & s_{k-2} & \cdots & 0 & 0 & 0 \\
m & \sum_{i=1}^{\infty} s_{k+i} & s_k & s_{k-1} & s_{k-2} & \cdots & 0 & 0 & 0 \\
m+1 & \sum_{i=2}^{\infty} s_{k+i} & s_{k+1} & s_k & s_{k-1} & \cdots & 0 & 0 & 0 \\
m+2 & \sum_{i=3}^{\infty} s_{k+i} & s_{k+2} & s_{k+1} & s_k & \cdots & 0 & 0 & 0 \\
\vdots & & & & & & & & \\
M-1 & \sum_{i=M-m}^{\infty} s_{k+i} & s_{k+M-m-1} & s_{k+M-m-2} & s_{k+M-m-3} & \cdots & s_k & s_{k-1} & \sum_{i=0}^{k-2} s_i \\
M & \sum_{i=M-m+1}^{\infty} s_{k+i} & s_{k+M-m} & s_{k+M-m-1} & s_{k+M-m-2} & \cdots & s_{k+1} & s_k & \sum_{i=0}^{k-1} s_i \\
Q & \sum_{i=M-m+1}^{\infty} s_{k+i} & s_{k+M-m} & s_{k+M-m-1} & s_{k+M-m-2} & \cdots & s_{k+1} & s_k & \sum_{i=0}^{k-1} s_i
\end{array}
$$

For practical applications, the performance of the model can be simply assessed by comparing the forecasted probabilities $P[E_t=Q]$ and $P[E_t=q]$ against the observed conveyor working frequency. However, since ships are bigger than trains, but assumed instantaneously loaded, assumption P5 may, in some situations, underestimate the lower side of the probability, $P[E_t=q]$. To correct this prospect the next model relaxes such assumption.

4.3 THE ONE-REFLECTING STATE MODEL

This last model generalizes the previous one by relaxing assumption P5. Thus, instead of infinite speed the conveyor performance will be restricted. Note that any definition of conveyor speed has to be vis-à-vis the train units which have been set to k units during the time interval unit T. Further, to

avoid the heavy notation that would result from adopting an arbitrary integer j to describe the conveyor performance during the same time interval T, it will be preferred to lose a bit of generality in favour of clarity and set the conveyor speed at j = 1 train units per time interval T. Later, in this section the important particular case k=j=1 will be examined. This particular case corresponds to have a perfect match between the conveyors capacity and the train arrival volume.

As before, the model only controls the stock level at the main storage. This stock can vary from the empty level to an upper level M. The overflow situation is represented by a reflecting barrier, state Q, one stock unit beyond M, which with probability one sends the system back to state M. On the other hand, the stock can be lower than m, or eventually reach zero, due to ship capacities and according to assumption P2. However, when the stock is below m the transition probabilities are affected by the action of the conveyor which, as discussed above, is able to resupply the main storage at a rate of one train unit per time interval T. Note again, as a consequence of only controlling the main stock, that the secondary stock level is assumed endless and also of unlimited capacity. The figure below illustrates the above description and to easy the notation the following simplification will be adopted:

$P[E_t=m_i]$ = Probability of the system to be i units below the lower stock level m, i=1,2,...m; thus, $m_m=0$; $m_{m-1}=1$, and so on.

---

0 1 ... m_2 m_1 m ... M Q

The resulting transition equations are:

$$P[E_{t+1}=Q] = \sum_{j=0}^{k-1} P[E_t=M-j] \sum_{i=j+1}^{k} s_{k-i} + P[E_t = Q] \sum_{i=1}^{k} s_{k-i}$$

$$P[E_{t+1}=M] = \sum_{j=0}^{k} P[E_t=M-j]s_{k-j} + P[E_t = Q]\ s_k$$

$$P[E_{t+1}=n] = \sum_{j=0}^{M-n} P[E_t=n+j]s_{k+j} + \sum_{j=1}^{k1} P[E_t=n-j]s_{k-j} + P[E_t = Q]\ s_{k+M-n}$$

for $m < n < M$ and $k1 = \min(k, n-m)$

$$P[E_{t+1}=m] = \sum_{j=0}^{M-m} P[E_t=m+j]s_{k+j} + \sum_{i=1}^{k+1} P[E_t=m_i]s_{k-i+1} + P[E_t = Q]\ s_{k+M-m}$$

$$P[E_{t+1}=m_i] = \sum_{j=0}^{M-m} P[E_t=m+j]s_{k+j+i} + \sum_{j=1}^{k1} P[E_t=m_j]s_{k+i+1-j} + P[E_t=Q]s_{k+M-m+i}$$

for $1 \leq i \leq m-1$ and $k1 = \min(k+i+1, m)$

$$P[E_{t+1}=0] = \sum_{j=0}^{M-m} P[E_t=m+j] \sum_{i=j}^{\infty} s_{k+m+i} + \sum_{j=1}^{m} P[E_t=m_j] \sum_{i=1}^{\infty} s_{k+m-j+i}$$

$$+ P[E_t = Q] \sum_{i=M}^{\infty} s_{k+i}$$

In the above model the probability of using the conveyor can be stated according to the following expressions:

$P[E_t = Q]$ = probability of using the conveyor in the away sense.

$\sum_{i=1}^{m} P[E_t=m_i]$ = probability of using the conveyor in the towards sense.

The transition equations given above are better seen as a transition matrix, shown on next page, in which it was supposed, for illustration purposes, that $k < m$ and $k < M-m$.

$$
P = \begin{array}{c|ccccccccc}
 & 0 & 1 & 2 & \cdots & m_1 & m & \cdots & M-1 & M & Q \\
0 & \sum_{I=1}^{\infty} s_{k+i} & s_k & s_{k-1} & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
1 & \sum_{I=2}^{\infty} s_{k+i} & s_{k+1} & s_k & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
2 & \sum_{I=3}^{\infty} s_{k+i} & s_{k+2} & s_{k+1} & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & & & & & & & & & & \\
m_1 & \sum_{I=m}^{\infty} s_{k+i} & s_{k+m-1} & s_{k+m-2} & \cdots & s_{k+1} & s_k & \cdots & 0 & 0 & 0 \\
m & \sum_{I=m}^{\infty} s_{k+i} & s_{k+m-1} & s_{k+m-2} & \cdots & s_{k+1} & s_k & \cdots & 0 & 0 & 0 \\
\vdots & & & & & & & & & & \\
M-1 & \sum_{I=M-1}^{\infty} s_{k+i} & s_{k+M-2} & s_{k+M-3} & \cdots & s_{k+M-m} & s_{k+M-m-1} & \cdots & s_k & s_{k-1} & \sum_{I=0}^{k-2} s_i \\
M & \sum_{I=M}^{\infty} s_{k+i} & s_{k+M-1} & s_{k+M-2} & \cdots & s_{k+M-m+1} & s_{k+M-m} & \cdots & s_{k+1} & s_k & \sum_{I=0}^{k-1} s_i \\
Q & \sum_{I=M}^{\infty} s_{k+i} & s_{k+M-1} & s_{k+M-2} & \cdots & s_{k+M-m+1} & s_{k+M-m} & \cdots & s_{k+1} & s_k & \sum_{I=0}^{k-1} s_i
\end{array}
$$

One particular case of the above model which has special interest is the one in which k=1, i.e., the speed of the conveyor is the same as the discharging volume arriving by the trains. For this important case, the transition matrix is shown below:

$$
P = \begin{array}{c|ccccccccccc}
 & 0 & 1 & 2 & \cdots & m_1 & m & \cdots & M-1 & M & Q \\
0 & 1-\sum_{i=0}^{1} s_i & s_1 & s_0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
1 & 1-\sum_{i=0}^{2} s_i & s_2 & s_1 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
2 & 1-\sum_{i=0}^{3} s_i & s_3 & s_2 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & & & & & & & & & & \\
m_1 & 1-\sum_{i=0}^{m_1} s_i & s_{m_1} & s_{m_1-1} & \cdots & s_2 & s_1 & \cdots & 0 & 0 & 0 \\
m & 1-\sum_{i=0}^{m} s_i & s_m & s_{m-1} & \cdots & s_2 & s_1 & \cdots & 0 & 0 & 0 \\
m+1 & 1-\sum_{i=0}^{m+1} s_i & s_{m+1} & s_m & \cdots & s_3 & s_2 & \cdots & 0 & 0 & 0 \\
\vdots & & & & & & & & & & \\
M-1 & 1-\sum_{i=0}^{M-1} s_i & s_{M-1} & s_{M-2} & \cdots & s_{M-m+1} & s_{M-m} & \cdots & s_1 & s_0 & 0 \\
M & 1-\sum_{i=0}^{M} s_i & s_M & s_{M-1} & \cdots & s_{M-m+2} & s_{M-m+1} & \cdots & s_2 & s_1 & s_0 \\
Q & 1-\sum_{i=0}^{M} s_i & s_M & s_{M-1} & \cdots & s_{M-m+2} & s_{M-m+1} & \cdots & s_2 & s_1 & s_0
\end{array}
$$

It is interesting to note that the discussion about the short-term decision process as presented at the end of chapter II could basically be reproduced here, since this third model allows similar evaluations leading to a decision making process quite analogous to those enumerated on section 2.4.

V. A PRACTICAL APPLICATION

This chapter discusses the application of the models introduced in Chapter IV to the same terminal, i.e., the Quebec Cartier iron ore terminal. Most of the data set remains unchanged, but of special importance now is the frequency of utilization of the conveyor as a function of the upper and lower stock control levels. Before presenting any result more information about the terminal and the way it operates has to be given.

The first important information regards the existence of a plant which transforms almost half of the incoming mineral into pellets to be eventually shipped by the same ships. Therefore, the data collection had to consider a revision of the train data and the identification of ships according to the type and amount of raw mineral exported in order to only consider the exported raw iron ore.

Regarding trains their average composition was derived from the plant production records which indicate that 47.3% of the mineral arriving to the terminal was transformed into pellets and, therefore, 52.7% was exported as raw material. Consequently, trains were reduced to almost half and the daily train arriving load was set to 21,900 ton, corresponding to a little over three times half trains. Certainly this is a simplification since the iron ore is not homogeneous and the load of the trains is not always nicely split into half to the plant and half for exportation. But in any case, this sort of information is not documented and there is no way to know the past data.

The depot system is composed of two sectors denominated main and secondary storage area. The raw material arriving by train is first driven to the main storage area. The iron ore directed to the plant goes immediately to

its own storage system and after processing it goes to separate depots through its own conveyors so that it can be entirely disregarded in the present study.

Regarding ships that export raw material, Table 5.1 gives their frequency distribution acording to size for every month of 1986. As noted before, classes correspond to multiples of 21,900 ton. The total number of ships is 91, an important decrease when compared to 200, the total number of ships using the terminal, as seen in Chapter III. The raw iron ore to be exported follows about the principles and assumptions adopted in Chapter IV,in particular section 4.3. In other words, there is an upper stock level M, M=24, corresponding to an approximate physical limitation. Whenever the stock goes beyond M, reaching a state denominated Q, Q=25, the conveyor transfers the excess to the secondary storage, whose stock level is not subject to control, i.e., it has infinite capacity. On the other hand, when the stock is below a certain value m, the conveyor resupplies iron ore from the secondary area. The existing relation between the speed of the resupply conveyor and the speed of the train discharging conveyor is the same as stated on the particular case of section 4.3, that is, k=j=1, while the speed of the conveyor which loads the ships is set to infinity, according to assumption P5. This setting is realistic since otherwise the ships would be directly charged from the secondary storage and m=0 would be a trivial solution.

Using a method entirely similar to that employed in Chapter III, the probability of total ship capacity (s_m, $m>0$) arrived during each time period was determined. The results are indicated in TABLE 5.2 below. The unit of capacity is 21,900 ton and the probabilities were truncated at s=12 and subsequently normalized.

Classes	Months												Total
	J	F	M	A	M	J	J	A	S	O	N	D	
0.5 - 1.5	2	0	0	3	0	3	0	1	0	1	0	2	12
1.5 - 2.5	2	1	4	0	1	1	2	3	2	0	3	3	22
2.5 - 3.5	1	1	0	0	3	0	1	1	1	2	0	0	10
3.5 - 4.5	1	2	2	3	3	0	2	0	1	1	1	2	18
4.5 - 5.5	3	2	0	3	0	1	3	1	2	2	2	1	20
5.5 - above	1	0	0	1	0	1	1	1	1	0	2	1	9
Total:	10	6	6	10	7	6	9	7	7	6	8	9	91

TABLE 5.1: Frequency Distribution of Ships According to Tonnage

s	probability	s	probability
0	.780169	7	.004408
1	.025649	8	.003419
2	.047446	9	.002795
3	.022926	10	.002252
4	.040620	11	.001248
5	.045238	12	.000385
6	.023346		

TABLE 5.2: Probability of Ship Tonnage

5.1 SOLUTION OF THE SYSTEM

The accumulation of stock, as noted for the previous model, is also observed here. Consider the ship capacity table given above. Its expected value is E[s] = .842, which is to be compared with the one train unit. As a consequence, there is a surplus of mineral which along the year results in an expected value measured in multiples of 21,900 ton given by:

$$(1 - E[s]) * 360 = 56.8$$

With the previous warning in mind, the one-reflecting state model discussed in section 4.3 was solved for quite a number of value combinations of Q and m, and these were:

$$Q = 20, 21, \ldots, 25$$

$$m = 1, 2, \ldots, 16$$

As for a sample example Figure 5.1 shows the results for the pair of values m=6 and Q=25. The referred figure indicates:

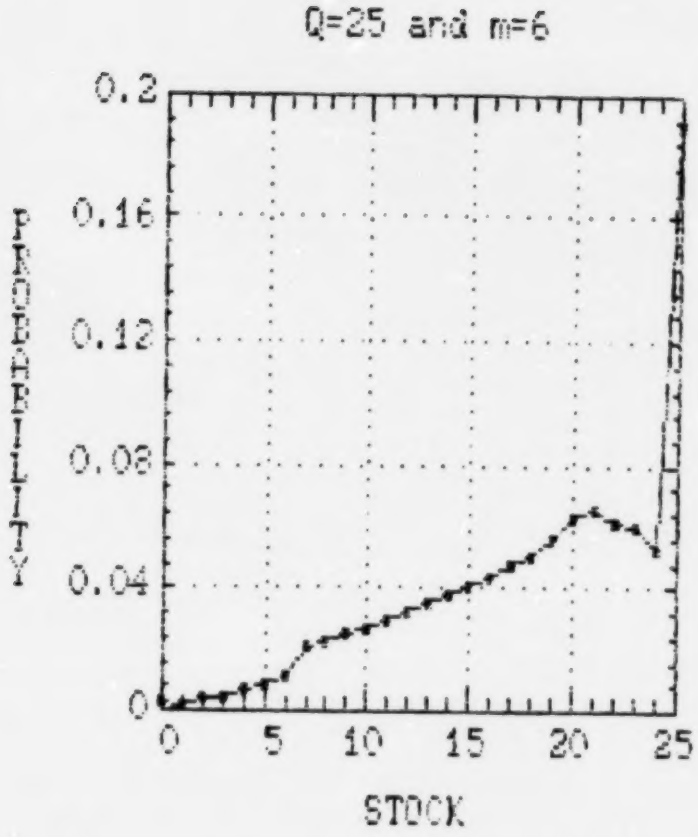


FIGURE 5.1: Probability Behaviour for m=6 and Q=25

$$P[E = Q] = .190064$$

$$\sum_{i=0}^{5} P[E = i] = .028969$$

$$P[E = Q] + \sum_{i=0}^{5} P[E = i] = .219033$$

The first probability, P[E = Q] = .190064, represents the probability of having the conveyor operating in the away sense, i.e., from the main to secondary storage. The second probability, .028969, is the sum of probabilities from 0 to m-1 and these represent situations in which the conveyor operates in the towards sense, i.e., from the secondary to main storage. Finally, the third probability represents the probability of making the conveyor operate in either way.

For the particular case being considered, i.e., k=j=1, every time the system reaches the state Q a complete train unit load, 21,900 ton is transferred to the secondary depot. As a consequence, the probability P[E=Q] can be easily converted into either hours of operation or expected volume moved by the conveyor. Somehow analogously, when the system reaches a state lower than m the conveyor starts ressuplying the main storage until the lower bound value m is attained. If a simplifying assumption is made that the conveyor will entirely ressuply the main storage without waiting for trains in the coming days to help it fill up the main storage, then computations become:

$$P[E = Q] * 21,900 = 4,162 \text{ ton}$$

$$\sum_{i=0}^{5} (6-i)\ P[E = i] * 21,900 = 1,734 \text{ ton}$$

Adding both volumes together gives a total of 5,896 ton. This number, when compared to 21,900 ton, gives 26.92% and this represents the average part of the train load which is displaced by conveyors in either sense.

Repeating the same computations for Q = 24 and m = 6, i.e., reducing by one unit the main storage capacity the corresponding results are:

$$P[E = 24] * 21,900 = .193486 * 21,900 = 4,237 \text{ ton}$$

$$\sum_{i=0}^{5} (6-i)\, P[E = i] * 21,900 = .087417 * 21,900 = 1,914 \text{ ton}$$

Adding both volumes together gives a total of 6,151 ton or 28.09% of the incoming mineral.

For an economic analysis, the two results above are quite important and they can be summarized as follows: For the given data set expressed by arrival rates of trains and ships, if the operating policy (m, M) is set to (6, 23) then 28.09% of the incoming mineral will suffer a dead movement between the main and the secondary storage or vice-versa; in contrast, if the operating policy is set to (6, 24) then 26.92% of the incoming mineral will be moved.

Transformed into costs, the savings in operation resulting from an increase in the main area are to be compared with the investment necessary to increase the same area. Also, alternative operating policies should be looked at before reaching the most economical trade-off between investment and operating costs.

Table 5.3 below gives a sample survey of a few operating policies and their effects on the percentage of the mineral that will be conveyed back and forth through the conveyor. Two key factors are pointed out by Table 5.3 both related to each one of the decision elements (m, M), and both transparent on last column.

First, consider M which represents fixed investment on ressources such as space and high capacity materials handling equipment. If both ressources are increased there is a resulting decrease on operating costs quantified by a reduction on the dead movement. In addition, the amount of reduction is an increasing function of m, the second decision element.

Consider now m and examine again Table 5.3. The use of the conveyor is an increasing function of m. The economic meaning of this relationship represe-

nts a comparison between two conflicting forces: one, the additional cost which results from increasing the conveyor use and second, the reduction in the risk and costs of delaying ships as a result of using a larger m as an operating policy.

Decision policy (m, M)	Volume main to secondary	Volume secondary to main	Total volume	Percentage of mineral conveyed
(5, 23)	4,173	1,509	5,682	25.9%
(5, 24)	4,105	1,309	5,414	25.0%
(6, 23)	4,237	1,914	6,151	28.09%
(6, 24)	4,162	1,734	5,896	26.92%
(11, 23)	4,270	4,227	8,998	41.09%
(11, 24)	4,631	3,783	8,414	38.4%

Table 5.3: Percentage of Conveyed Mineral for Selected Types of Operating Policies

5.2 THE CONVEYOR OPERATION

This section extends the previous discussion on the responsiveness of the conveyor to variations of Q and m by reporting two other experiments that have been made. The first makes simultaneous variations on Q and m to derive the interesting conclusion that the conveyor operating probability depends more on the difference Q-m than on the absolute values Q or m. The second experiment fixes Q and examines a larger spectrum of m values. Note that for both cases a straight addition of steady state probabilities was adopted. This contrasts with the procedure used on the previous section where lower than m probabilit-

ies were weighted. Both procedures differ on the assumptions used but the one adopted on the previous section is certainly preferable when necessary to transform into tonnagesconveyed while the straight addition of probabilities is a better way to simply measure the probability of using the conveyor.

Table 5.4 makes a summary of the experience of simultaneously changing both decision policy elements Q and m. Thus, row entries indicate the differences Q-m for a few values in the range 9 to 19. Column entries fix consecutive values of Q ranging from 20 to 25. The table values give the probability of using the conveyor in both ways for the corresponding Q-m, Q, and, consequently, m. If the table is looked by column it can be stated that the probability of using the conveyor increases as the value of the difference Q-m decreases. If the table is looked by row it can be noticed that all elementss along each row are remarkably similar and this leads to the conclusion that in determining the probability of using the conveyor what counts are the differences betwen Q and m rather than Q or m.

Figure 5.2 sets Q=25 and varies m from 1 to 16, The figure indicates graphically the probability of using the conveyor in both the away or towards sense, and the total probability of using the conveyor. As expected, all probabilities increase quite fast according to increasing values of m. In words, any increase in m to avoid the risk of delaying a ship by lack of stock in the main storage system imposes an increase in operating costs.

Q-m	Q = 20	Q = 21	Q = 22	Q = 23	Q = 24	Q = 25
19	.2084	.2085	.2150	.2158	.2184	.2190
18	.2139	.2210	.2220	.2248	.2255	.2267
13	.2752	.2758	.2767	.2770	.2774	.2775
12	.2902	.2912	.2915	.2919	.2921	.2923
10	.3283	.3288	.3290	.3293	.3294	.3295
9	.3526	.3528	.3531	.3532	.3533	.3534

TABLE 5.4: Probability of Using the Conveyor as a Function of m and Q-m

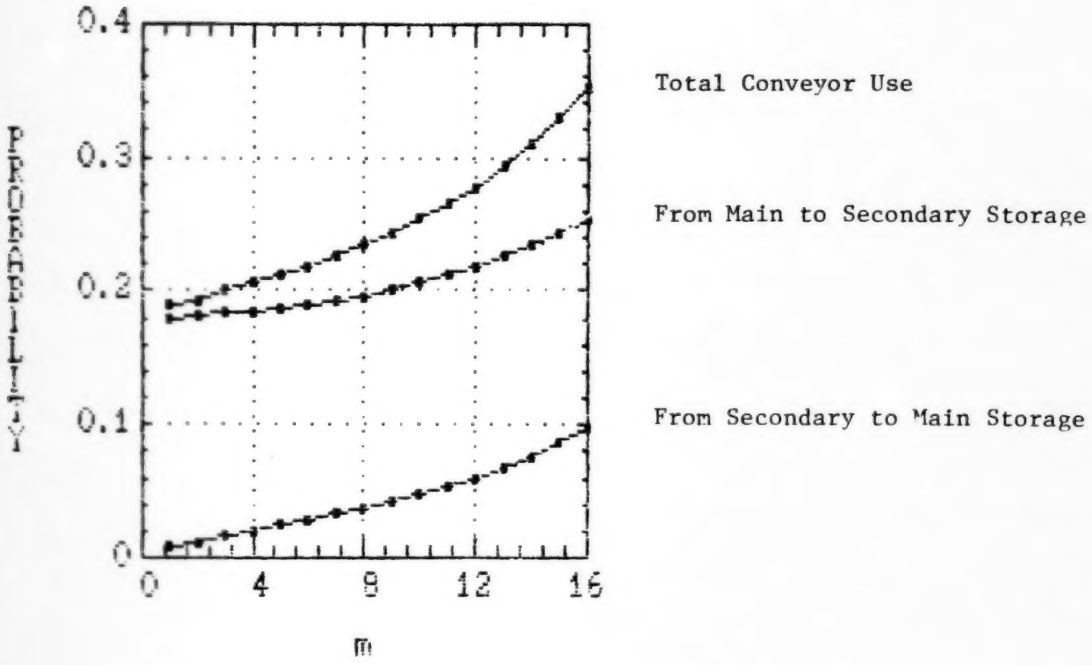


FIGURE 5.2: Probability of Using the Conveyor as a Function of m

5.3 CONCLUSIONS

The present section summarizes some of the findings for the case a terminal is formally designed or practically operated in the form described in Chapter IV.

Obviously, if investment is to be kept reasonably low, no high capacity recuperation equipment is to cover an infinite area but rather its action has to be restricted to a few hundred square meters. The rest of the storage area is to be covered by lower cost materials handling equipment.

Chapters IV and V provide fundamental elements to either designing a terminal with two levels of storage differentiated by space, cost, and technical equipments or also to operate such an existing terminal.

In summary, given the arrival rates for trains and ships and checked its independence, the steady state probabilities can be found by using one of the models proposed on Chapter IV. If the model introduced on section 4.3 is used, basic information for evaluating the trade-off between investment costs and operating costs is derived. The basic information referred assumes the form of volumes transferred between the two storage areas, risks of running out of stock or delaying a ship, and the proper dimensions of the main storage area. Applying cost data to the elements provided by the model the most economical terminal design can be developed.

Once an (m, M) operating policy established the larger the upper level M the lesser the conveyor utilization; the larger the lower level m more frequent the use of the conveyor. However, if both parameters vary simultaneouusly the use of the conveyor, or transfers between both storage areas tend to stay stable once the difference Q-m is constant showing limited dependence on the values of Q or m.

Nothing has been said in this chapter about the short term decision process because the main focus here was the design of the terminal and the relation between investment and operating costs. It seems transparent that the short term may well require the instruments discussed on chapters II and III.

BIBLIOGRAPHY

[1] Ghosal, A. Some Aspects of Queueing and Storage Systens. Lecture Notes in Economics and Mathematical Systems 23. Springer Verlag, New York, 1970.

[2] Moran, P. A. P. The Theory of Storage. Methuen, London, 1959.

[3] Pizzolato, N. D. and O. C. Videira. Um Modelo Markoviano para Dimensionar Estoques em Terminais Maritimos. Pesquisa Operacional, Vol. 5, no. 2, dezembro 1985.

[4] Prabhu, N. U. Queues and Inventories - A Study of Their Basic Stochastic Processes, Wiley, New York, 1965.

[5] Hadley, G. and T. M. Whitin. Analysis of Inventory Systems, Prentice Hall, 1963.

REMERCIEMENTS

Les auteurs tiennent à remercier plusieurs personnes pour leur participation à la réalisation de ce travail. Ainsi MM. François Soumis, Michel Gendreau et Warren Powell ont contribué à différents niveaux et Mlle Lucie Bibeau s'est chargée des travaux sur ordinateur.